Exam 1 Solutions

October 16, 2010

1 Problem 1

a) Is shown in Fig. 1

\[ v(t) = \frac{dx}{dt} = (20 \text{ m/s}) - (20 \text{ m/s})e^{-t/(2s)} \]  \hspace{1cm} (1)

b) \[
\begin{align*}
\frac{dv}{dt} &= (10 \text{ m/s}^2)e^{-t/(2s)} \\
\frac{da}{dt} &= (10 \text{ m/s}^2)e^{-t/(2s)}
\end{align*}
\]  \hspace{1cm} (2)

c) Yes, and the force changes with time,

\[ F(t) = ma(t) = 10N e^{-t/2s} = -\frac{1}{2}v(t)(Ns/m) + 10N \]  \hspace{1cm} (3)

So there is a constant force (10N) in the direction of motion and a resistive force proportional to velocity in the opposite direction. This could have been a description of an object falling from height \( h = x(0) = 10 \text{ m} \) with \( x \) increasing (positive) towards the ground. Right after the object is released \( v(t \sim 0) \sim (20 \text{ m/s}) - (20 \text{ m/s})(1 - t/(2s)) \sim 10(\text{m/s}^2)t \) starts at zero at \( t = 0 \) and slowly increases with acceleration \( 10\text{m/s}^2 \). This corresponds to the constant force of 10N aka gravity \( mg = 1kgg \sim 10N \) in the last term in Eq. 3. And the other one \(-1/2v(t)(Ns/m)\) would be the drag force.

2 Problem 2

a) \[
\begin{align*}
v_A(t) &= (2m/s)i - (10tm/s^2)j \\
v_B &= (1m/s) \cos(30^0)i + [(1m/s) \sin(30^0) - 10tm/s^2]j = (0.87m/s)i + (0.5m/s - 10tm/s^2)j
\end{align*}
\]  \hspace{1cm} (4)
Figure 1: Vertical axis is \( x \) in meters, horizontal is time in seconds

b )

\[
A(t) = (2m/st)i + (2m - 5t^2m/s^2)j \\
B(t) = (0.87tm/s)i + (0.5tm/s - 5t^2m/s^2)j
\]

(5)

c )

\[
C(t) = A(t) - B(t) = (1.13tm/s)i + (2m - 0.5tm/s)j
\]

(6)

3 Problem 3

a ) Is shown in Fig. 2 Since both blocks are not moving in the vertical directions magnitudes of the vertical forces satisfy the following relations for block \( m \)

\[
N = F_m = mg
\]

(7)

for block \( M \)

\[
R = F_M + N = Mg + mg
\]

(8)

c) Take + horizontal direction to coincide with the direction of the applied force. For the upper block we have

\[
F - F_f = ma_m
\]

(9)

where \( F_f \) is the kinetic friction that block \( M \) is opposing the motion of block \( m \) Since the normal force between the blocks is \( N = mg \) thus \( F_f = \mu mg \) we find

\[
F - mg\mu = ma_m
\]

(10)
Top block (block m):
- \( F_m \): Gravity, originates from the Earth acts on block m
- \( N \): Normal reaction from block M, originates from block M block acts on block m
- \( F \): Applied external force acts on block m
- \( F_f \): Kinetic friction, originates from block M acts on block m

Bottom block (block M):
- \( F_M \): Gravity, originates from the Earth acts on block M
- \( -N \): Normal reaction from block m, originates from block m block acts on block M
- \( R \): Normal reaction from the table, originates from the table acts on block M
- \( -F_f \): Kinetic friction, originates from block m acts on block M

Figure 2:
so the acceleration of the upper block is given by

\[ a_m = \frac{F}{m} - \mu g \]  

(11)

From Newton’s third law we conclude that the lower block is pushed forward (in direction of \( F \)) with force \( F_f \) (see figure). So for the lower block we find

\[ F_f = Ma_M \]  

(12)

or

\[ a_M = \mu g \frac{m}{M} \]  

(13)

4 Problem 4

Before the middle bad is cut the lower ball has a tension \( T_1 \) up and gravity \( m_1 g \) down, and being at rest

\[ T_1 = m_1 g \]  

(14)

The upper ball has tension \( T_2 \) up and \( T_1 \) down and gravity \( m_2 g \) down

\[ T_2 = T_1 + m_2 g = (m_1 + m_2)g \]  

(15)

Right after the band is cut tension \( T_1 \) disappears right away but not \( T_2 \) (elastic band) so that the lower ball with have acceleration \( a_1 = g \) down and the upper ball

\[ T_2 - m_2 g = m_2 a_2 \]  

(16)

or

\[ m_1 g = m_2 a_2 \]  

(17)

\[ a_2 = \frac{(m_1/m_2)g}{(1/3)g} \]