Exam 1 Solutions

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1 Problem 2

Initial energy
\[ E_i = K_i + U_i \] (1)

Final energy
\[ E_f = K_f + U_f \] (2)

Energy difference is equal to work done by all other forces that are not included in the potential energy. In this case this is friction,
\[ E_i - E_f = \text{Work done by friction} \] (3)

Thus
\[
\text{Work done by friction} = K_i + U_i - K_f - U_f = \frac{m(v_i^2 - v_f^2)}{2} = \frac{0.020kg[40000 - 2500]m^2/s^2}{2} = 375J
\] (4)

Since potential energy depends only on location, and at the beginning and at the end height is the same so \( U_i = U_f \).

2 Problem 2

The minimum energy required is equal to the difference in potential energy:
\[ \Delta U = m_w g \Delta h = (V \rho_w) g H \] (5)

So minimal power is (using \( g = 10m/s^2 \))
\[
P = \frac{V \rho_w g H}{t} = \frac{(0.1m)^31000kg/m^310m/s^20.2m}{60s} = 0.033W
\] (6)
3  Problem 3

Measured with respect to the middle of the square a

\[ x_{cm} = \frac{1}{10m} \left[ m(-a/2) + 3m(-a/2) + 2m(a/2) + 4m(a/2) \right] = \frac{a}{10} = 1\text{cm} \]
\[ y_{cm} = \frac{1}{10m} \left[ m(a/2) + 3m(-a/2) + 2m(a/2) + 4m(-a/2) \right] = -\frac{a}{5} = -2\text{cm} \]  

(7)

b Since none of the masses have x-component of velocity \( v_{x;cm} = 0 \) for the y-component of velocity we

\[ v_{y;cm} = \frac{1}{10m} (mv_y) = \frac{v_y}{10} = 0.1\text{m/s} \]  

(8)

4  Problem 4

a) Moment of inertia is the sum of moments of the hoop and the bars

\[ I = MR^2 + 2 \left( \frac{1}{12} mL^2 \right) = MR^2 + \frac{1}{6} mL^2 \]
\[ = 1 \times 1^2 kgm^2 + \frac{1}{6} \times 1^2 kgm^2 = 2kgm^2 \]  

(9)

b) If \( F \) was the only horizontal force then for the horizontal acceleration of the center of mass we would have

\[ F = (M + 2m)a_{cm} \]  

(10)

or

\[ a_{cm} = \frac{F}{M + 2m} = \frac{1N}{13kg} = \frac{1}{13} m/s^2 = 0.077 m/s^2 \]  

(11)

and for the angular acceleration around the center of mass

\[ \alpha = \frac{\tau}{I} = \frac{RF}{I} = \frac{1mN}{2kgm^2} = 0.51/s^2 \]  

(12)

c.) So the rolling condition \( \alpha = a_{cm}/R \) is not satisfied

d) Thus there must be another force to have rolling. The only possibility is friction – call it \( f \). Assume it acts in the same direction as \( F \). Then for the center of mass we would have

\[ F + f = (M + 2m)a_{cm} \]  

(13)

and from angular rotation around the center of mass.

\[ FR - fR = I\alpha \]  

(14)
Note the sign of the torque from $F$ (positive) has been chosen such that positive $a_{cm}$ corresponds to positive $\alpha$. Together with the rolling condition $\alpha = a_{cm}/R$ these two equations give

$$2F = (M + 2m + \frac{I}{R^2})a_{cm} \quad (15)$$

so

$$a_{cm} = \frac{2N}{(1kg + 12kg + 2kg)} = \frac{2}{15} m/s^2 \quad (16)$$

and the frictional force

$$2f = (M + 2m - \frac{I}{R^2})a_{cm} \quad (17)$$

$$f = \frac{(1kg + 12kg - 2kg)(2/15)m/s^2}{2} = \frac{11}{15} N \quad (18)$$

and it is in the same direction to $F$.

Repeating for hoop without bars

a)

$$I = MR^2 = 1kgm^2 \quad (19)$$

b) If $F$ was the only horizontal force then for the center of mass

$$F = Ma_{cm} \quad (20)$$

gives

$$a_{cm} = \frac{1N}{1kg} = 1m/s^2 \quad (21)$$

and for the angular acceleration around the center of mass

$$\alpha = \frac{\tau}{I} = \frac{RF}{I} = \frac{1mN}{1kgm^2} = 1s^{-2} \quad (22)$$

c) So in this case the rolling condition $\alpha = a_{cm}/R$ is satisfied and there is no need to look for any other horizontal force.

d) In this case rolling occurs without any friction. (The force of friction is 0)